Gross Capital Flows and the Returns to Capital

Enisse Kharroubi*

Preliminary and Incomplete

First Version: September 2008 This version: December 2008

Abstract

This paper builds a two region world economy in which domestic and foreign capital are complements to each other. This framework provides a simple and rich framework to study the taxonomy of international capital flows and their macroeconomic implications. We show in particular that the equilibrium of the global capital market can be such that countries import and export capital at the same time. In this case, we derive three properties. First there can be multiple equilibria in the size and the direction of capital flows, thereby providing some theoretical underpinnings to episodes of capital flow reversals. Second when the equilibrium is unique, countries with larger capital markets are more likely every thing else equal to be net capital importers. Finally countries with more efficient financial markets are shown to run a net positive return on external assets compared to external liabilities.

Keywords: Cross-border capital flows, borrowing constraints, exorbitant privilege, return on capital.

JEL Classification: D82, E44, F36, G15, G21, O16.

* Banque de France. Address: 1, rue de la Vrillière 75049 Paris cedex 01. e-mail: first name.surname(at)banque-france.fr. The views expressed herein are those of the author and do not necessarily reflect those of Banque de France or the Eurosystem. Usual disclaimers apply.
1 Introduction

Over the last years, the inter-temporal approach of the current account has been significantly enriched. Theories that have contributed in this field have essentially highlighted the "equilibrium" properties that lie behind what is now well-known as the phenomenon of global imbalances. In this extended framework, financial development broadly understood as the ability to generate financial assets for investment plays a crucial role in determining the direction and the size and the return on international capital flows. This paper is an attempt to contribute to this literature by building a simple general equilibrium model with two countries where financial market imperfections are devoted a particular attention. We focus on one particular aspect: namely the existence of some complementarity for borrowers between domestic capital and the access to finance from abroad and we show it to provide a simple framework that is able to replicate a number of key stylized facts of the literature on global imbalances.

1.1 The model, the results

We consider a general equilibrium model with two economies (Home and Foreign) populated with entrepreneurs (borrowers) and investors. Entrepreneurs have the best opportunities in each region so that they have incentives to borrow from investors located in the same region and investors located abroad. Cross-border capital flows are therefore linked to the volume of capital entrepreneurs borrow from investors located abroad. In principle given that investors provide the same service to entrepreneurs, it is likely that entrepreneurs borrow from investors of their own region so that there is no reason why capital would flow across borders. In the model, we break down this possibility by making the two core assumptions of the paper. First domestic borrowing and borrowing from abroad are complements. Typically, the capital an entrepreneur can borrow from investors located abroad increases with the capital borrowed from domestic investors on the domestic capital market. Second we introduce transactions costs so that the marginal return to capital depends on its source -the lending efficiency of the investor- as well as on its destination -the borrowing

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1To simplify the exposition of the model, we restrict our attention to the case where investors do only lend capital but do not borrow.
efficiency of the entrepreneur-. In this framework, we first characterize the taxonomy of international capital flows. The equilibrium of the global capital market can either feature no cross-border capital flows, one way capital flows or two way capital flows, each region importing and exporting capital. This depends on the ratio in entrepreneurs’ productivity across regions. A region is more likely to be a capital importer when entrepreneurs in this region are more productive. Moreover the situation where entrepreneurs productivity is relatively similar may either exhibit no cross-border capital flows or exhibit two way capital flows, i.e. both regions import and export capital. This typically depends on transactions costs on the capital market. When investors lending efficiency is positively related to entrepreneurs borrowing efficiency across regions, then capital flows in both directions when entrepreneurs’ productivities are relatively similar. On the contrary if investors lending efficiency is negatively related to entrepreneurs borrowing efficiency across regions, then capital does not flow across regions when entrepreneurs’ productivities are relatively similar and financial autarky is an equilibrium. To put it in a nutshell, it is necessary that one region dominates the other in terms of financial efficiency to obtain an equilibrium of the global capital market where the two regions tend to import and export capital.

Secondly the paper looks at the salient properties of the equilibrium with two way capital flows. We provide three of them. First there are multiple equilibria in the global capital market when complementarity between domestic and foreign capital sources is sufficiently large in each region. This means that at the equilibrium of the global capital market, capital can either flow from Home to Foreign, or from Foreign to Home or in both directions. The mechanism for this result is as follows. When borrowing from abroad becomes profitable for entrepreneurs in a given region, the cost they are willing to pay for domestic capital increases because of the positive impact of domestic borrowing on the access to foreign capital. As a consequence if capital flows to the Home region for instance, the cost of capital charged by investors of the Home region increases which precludes entrepreneurs of the Foreign region to borrow from investors of the Home region. As a result, capital effectively flows to the Home region. The same can happen if capital flows to the Foreign region. Finally capital can also flow in both directions because a larger cost of capital borrowed from abroad tends to reduce the cost of domestic capital. Second, we derive -in the case of the
two-way capital flows-the size of equilibrium cross-border capital flows. We show in particular that an increase in the size of domestic capital markets always tends to deteriorate the current account balance. Every thing else equal, in particular financial efficiency and entrepreneurs’ productivity-regions where the financial sector is larger-investors are more wealthy-are more likely to be net capital importers. On the contrary, regions where the corporate sector is larger-entrepreneurs are more wealthy-are more likely to be net capital exporters. While a priori counter-intuitive, this result is fairly straightforward to understand once complementarity is taken into account. If the domestic financial sector is larger, then entrepreneurs can borrow more capital domestically and hence can borrow more from abroad. As a consequence, entrepreneurs in the other region need to reduce domestic borrowing and borrowing from abroad given the complementarity. A large (resp. small) domestic financial sector is therefore associated with a current account deficit (resp. surplus). Third, we look at the cost of capital investors of each region charge at the equilibrium of the global capital market. Given the existence of borrowing constraints, the cost of capital charged by investors of region will typically differ. When capital flows in both directions, a region always runs a income surplus (a positive difference between the return on foreign assets and the cost of foreign liabilities) when domestic investors are more efficient in lending than investors located abroad. Put differently, in the presence of complementarity between domestic and foreign capital, the efficiency of domestic financial markets tends to materialize into a positive difference between the return on foreign assets and the cost of foreign liabilities.

1.2 Related literature

Without paying justice to the entire literature, it possible to highlight two different approaches in the theoretical literature on the financial causes of global imbalances. The first approach is based on the asset shortage view: Caballero, Farhi and Gourinchas (2008) show that global imbalances-along with the decline in long run real rates and the rise in the share of US assets in global portfolio – can be rationalized in a model where regions differ in growth potential and in the ability to generate financial assets. Regions with high growth potential but low ability to generate financial assets will typically be capital exporters because they lack the financial instruments to recycle their savings. On the contrary regions with high growth
potential and high ability to generate financial assets will be capital importers as they will provide financial instruments to countries with a low ability to generate financial assets. Related to the asset shortage view is the bypass effect formalized in Ju and Wei (2007). They propose a framework where financial openness help dampen the negative consequences of an inefficient domestic financial system. Two-way capital flows can act to by-pass the inefficiency of the domestic financial system, domestic savings leaving the country in the form of financial capital outflows but domestic investment takes place via inward foreign direct investment.

A second strand of literature focuses on the incompleteness of financial market from a risk diversification perspective. Countries that lack the ability to diversify risks domestically will tend to export capital as to insure domestic consumption against large fluctuations. Mendoza, Quadrini and Victor Rios-Rull (2007) provide a precautionary savings model with heterogenous agents who face non-insurable idiosyncratic risk due to asset market incompleteness. When a country with low financial development integrates with a country with higher financial development, it turns out that agents located in the former country are willing to lend to agents located in the latter because this helps them smooth their consumption, especially when they face a bad shock. Similarly, Miller and Zhang propose a model where agents face loss aversion. With pronounced loss aversion in emerging markets, they show that precautionary savings can generate substantial net capital flows from emerging markets towards mature countries. However they also note that this phenomenon is likely to be temporary as it should come to an end when adequate reserve levels are achieved.

Empirically, the literature has looked for evidence of the impact of financial institutions on current account balance and net foreign asset position. Forbes (2008) for instance provides evidence that the lack of domestic financial development is a significant factor affecting the decision of non resident agents to invest in the US. Ito and Chinn (2007) show that financial openness is associated with a smaller current account balance in developing countries. Similarly, they show that increases in the size of financial markets tends to induce a decline in the current account balance in industrial countries. Finally a debate has risen on the question of whether the returns of external assets and liabilities is systematically different, especially in the case of the US, given that a significant net return on foreign assets would be a source of stability in the presence of a large current account deficit. Basically Gourinchas and Rey (2008) have argued in favor of the
existence of a significant net return in favor of the US when comparing their external assets and liabilities while Curcuru et al. (2007) make the point that such a claim does not hold outside the class of direct investment.

The paper is organized as follows. Section 2 lays down the main assumptions of the model. In section 3, we derive the equilibrium of the global capital market highlighting the different possible equilibria of the global capital market. Section 4 derives the main properties of the equilibrium with two-way capital flows. Conclusions are finally drawn in section 5.
2 Preferences and technologies

We consider a world economy with two regions, Home and Foreign. In both regions, there is an equal number of entrepreneurs and investors. Entrepreneurs’ initial capital is normalized to 1 and investors’ initial capital is \( k \). The marginal return to capital for entrepreneurs of the Home region (resp. Foreign region) is \( R_h \) (resp. \( R_f \)). Entrepreneurs can borrow from investors of their own region and/or from investors of the other region. Investors can supply capital to entrepreneurs of their own region and/or to entrepreneurs of the other region. Entrepreneurs are not constrained in the volume of capital they borrow from investors of their own region. However they face a constraint on the volume of capital they can borrow from investors of the other region. These borrowing constraints are detailed in the next section.

2.1 Borrowing constraints

Entrepreneurs face two problems on the capital market. First financial transactions are costly. Second financial contracts are not perfectly enforceable: entrepreneurs can default strategically on their liabilities. Consider an entrepreneur of region \( i \) with one unit of own capital (equity) who borrows \( d_i \) units of capital from investors of region \( i \), and \( d_{i,j} \) units of capital from investors of region \( j \). Transaction costs are as follows: an entrepreneur of region \( i \) can effectively invest a fraction \( \tau_i \) of capital borrowed from investors of region \( i \) \((0 \leq \tau_i \leq 1)\). Similarly, an entrepreneur of region \( i \) can effectively invest a fraction \( \tau_{i,j} \) of capital borrowed from investors of region \( j \) \((0 \leq \tau_{i,j} \leq 1)\). The entrepreneur’s total investment is therefore equal to \( 1 + \tau_i d_i + \tau_{i,j} d_{i,j} \) and the profit the entrepreneur of region \( i \) enjoys when it does not default writes as

\[
\pi = (1 + \tau_i d_i + \tau_{i,j} d_{i,j}) R_i - r_i d_i - r_{i,j} d_{i,j}
\]

\( r_i \) being the gross interest rate investors of region \( i \) charge to their borrowers.\(^2\) When the entrepreneur defaults, it pays back only a given fraction of his liabilities but this has a cost: the return on investment is

\(^2\)Note that a lender in a given region charges a unique cost of capital irrespective of the origin of the borrower.
lower. The entrepreneur’s profit in this case writes as

$$\pi_1 = (1 + \tau_i d_i + \tau_{i,j} d_{i,j}) (R_i - \sigma) - q_i r_i d_i - q_{i,j} r_{i,j} d_{i,j}$$

(2)

where $\sigma$ is the marginal cost to default, $q_i$ (resp. $q_{i,j}$) is the proportion liabilities, investors of region $i$ (resp. region $j$) are able to recoup when an entrepreneur of region $i$ defaults. To recover a fraction $p$ of liabilities of size $d$ extended to an entrepreneur of region $i$, a investor of region $i$ (resp. of region $j$) needs to pay $c_i \ln \left( \frac{1}{1-p} \right) d$ (resp. $c_{i,j} \ln \left( \frac{1}{1-p} \right) d$). investors in each region therefore determine the fractions $q_i$ and $q_{i,j}$ as to maximize their income net of recovering costs

$$q_i = \arg\max_p p r_i d_i - c_i \ln \left( \frac{1}{1-p} \right) d_i$$

$$q_{i,j} = \arg\max_p p r_{i,j} d_{i,j} - c_{i,j} \ln \left( \frac{1}{1-p} \right) d_{i,j}$$

Normalizing $\sigma$ to 1, we then make the core assumption of the paper: the parameters $c_i$ and $c_{i,j}$ are such that

$$c_i < \tau_i \text{ and } c_{i,j} > \tau_{i,j}$$

(3)

The following proposition can then be derived.\(^3\)

**Proposition 1** Denoting $\lambda_i = \frac{1}{c_{i,j} - \tau_{i,j}}$ and $\theta_i = \frac{\tau_i - c_i}{c_{i,j} - \tau_{i,j}}$, an entrepreneur of region $i$ with one unit of capital pays back its liabilities if and only if the volume of capital $d_{i,j}$ it borrows from investors of region $j$ verifies

$$d_{i,j} \leq \lambda_i + \theta_i d_i$$

(4)

\(^3\)Under this assumption, when $p = 0$, the marginal cost for lenders of region $i$ to recover their assets from entrepreneurs of region $i$ ($c_i$) is lower than the marginal cost $\tau_i$ for entrepreneurs of region $i$ to default. On the contrary, when $p = 0$, the marginal cost for lenders of region $j$ to recover their assets from entrepreneurs of region $i$, ($c_{i,j}$) is larger than the marginal cost for entrepreneurs of region $i$ to default $\tau_{i,j}$. 

8
**Proof.** Let us focus on the case of an entrepreneur of the Home region. When it defaults, investors of the Home and the Foreign region choose respectively $q_h$ and $q_f$ such that

$$c_h = (1 - q_h) r_h \text{ and } c_{h,f} = (1 - q_{h,f}) r_f$$

(5)

Plugging these equalities in the incentive constraint $\pi \geq \pi_1$ we easily end up with

$$1 + \tau_h d_h + \tau_{h,f} d_{h,f} \geq c_h d_h + c_{h,f} d_{h,f}$$

(6)

which simplifies as (4) for $i = h$ and $j = f$. A similar argument can be made in the case of an entrepreneur of the Foreign region. ■

Under the assumptions $c_i < \tau_i$ and $c_{i,j} > \tau_{i,j}$, the parameter $\theta_i$ is always positive. The volume of capital an entrepreneur can borrow from abroad increases with the volume of capital it borrows domestically. This is due to the difference in investors’ efficiency to recover their claims from entrepreneurs who default. Because a investor is very efficient in recovering domestic claims, i.e. claims on entrepreneurs located in the same region, investors of the other region can increase their capital supply when entrepreneurs increase domestic borrowing without raising incentives to default.

### 2.2 The capital demand

Given these credit constraints, we can now determine the demand for capital on the capital market from entrepreneurs. To do so, let us consider an entrepreneur of region $i$. Its problem consists in choosing the debt portfolio that maximizes its profit conditional on the borrowing constraint it faces.

$$\max_{d_i,d_{i,j}} (1 + \tau_i d_i + \tau_{i,j} d_{i,j}) R_i - r_i d_i - r_{j,i} d_{i,j}$$

$$d_{i,j} \leq \lambda_i + \theta_i d_i$$

Entrepreneurs’ optimal demand for capital is then as follows...
Proposition 2  The demand for capital $d_{i,j}^*$ from entrepreneurs of region $i$ to investors of region $j$ is

$$d_{i,j}^* = (\lambda_i + \theta_i d_{i}^*) 1 [r_j \leq \tau_{i,j} R_i]$$

(7)

where $1[x] = 1$ if $x$ is true and 0 otherwise. The demand for capital $d_i^*$ from entrepreneurs of region $i$ to investors of region $i$

$$d_i^* = \begin{cases} +\infty & \text{if } r_i \leq \tau_i R_i + \theta_i (\tau_{i,j} R_i - r_j)^+ \\ 0 & \text{otherwise} \end{cases}$$

(8)

where $(x)^+ = \max(x; 0)$

Proof. cf. appendix □

When entrepreneurs cannot borrow from abroad, then the cost of capital $r_i$ at which their demand for domestic capital becomes zero is equal to $\tau_i R_i$. However when entrepreneurs can borrow profitably from abroad, i.e. $\tau_{i,j} R_i > r_j$, then the cost of capital $r_i$ at which the demand for domestic capital becomes zero increases above $\tau_i R_i$. When the economy opens to foreign capital, entrepreneurs are therefore willing to borrow capital from domestic investors at a larger cost. Put differently, financial openness moves the demand for domestic capital to the right when domestic and foreign capital are complements: for a given interest rate, demand for domestic capital is always larger in the open economy. Denoting $k_i$ is the aggregate demand for capital from entrepreneurs of region $i$ to investors of region $i$, the aggregate demand for capital $k_{i,j}$ from entrepreneurs of region $i$ to investors of region $j$, writes as

$$k_{i,j} = (\lambda_i + \theta_i k_i) 1 [r_j \leq \tau_{i,j} R_i]$$
2.3 The capital supply

The capital supply comes from investors. Their capital supply \( k \) is totally inelastic. The equilibrium of the capital market in region \( i \) and \( j \) therefore write as

\[
k_i + (\lambda_j + \theta_j k_j) 1 [\tau_{j,j} R_j \geq r_i] = k
\]

\[
k_j + (\lambda_i + \theta_i k_i) 1 [\tau_{i,j} R_i \geq r_j] = k
\]

In what follows we make the necessary restrictions on parameters \( \lambda, \theta \) and \( k \) so that the volume of capital borrowed domestically in each region \( k_h \) and \( k_f \) as well as the volume of cross-border capital flows \( k_{h,f} \) and \( k_{f,h} \) cannot be strictly negative. Denoting \( \mu_i = \lambda_i + \theta_i k \) the maximum amount of capital entrepreneurs from region \( i \) can borrow from investors of region \( j \), we assume that for \( i = \{h, f\} \), \( \mu_i \leq k \). To complete the description of the equilibrium of the global capital market, we need to determine the equilibrium costs of capital \( r_i \) and \( r_j \) investors of region \( i \) and investors of region \( j \) charge to their borrowers. Given the results of proposition , if entrepreneurs of region \( i \) do not borrow from investors of region \( j \), then the equilibrium cost of capital investors of region \( i \) charge to their borrowers is \( r_i = \tau_i R_i \). On the contrary if entrepreneurs of region \( i \) do borrow from investors of region \( j \), then the equilibrium cost of capital investors of region \( i \) charge to their borrowers is \( r_i = \tau_i R_i + \theta_i (\tau_{i,j} R_i - r_j) \). With a similar argument for the equilibrium cost of capital \( r_j \) charged by investors of region \( j \) we have

\[
r_i = \tau_i R_i + \theta_i (\tau_{i,j} R_i - r_j)^+
\]

\[
r_j = \tau_j R_j + \theta_j (\tau_{j,i} R_j - r_i)^+
\]

The equilibrium of the global capital market is therefore completely described by the systems (9) and (10).
3 Equilibrium of the global capital market

A priori, there are four possible configurations of the global capital market: capital can flow (i) from the Home to the Foreign region, (ii) from the Foreign to the Home region, (iii) in both directions, i.e. from Home to Foreign and vice-versa, or (iv) capital does not flow across borders.

3.1 The no capital flow equilibrium

Let us begin by considering the case where capital does not flow across regions. This situation is an equilibrium if and only if entrepreneurs of the Home region prefer not to borrow from investors of the Foreign region and entrepreneurs of the Foreign region prefer not to borrow from investors of the Home region. This holds when two conditions are satisfied: (i) entrepreneurs of the Home region make losses on capital borrowed from investors of the Foreign region: \( \tau_{h,f} R_h < r_f \) and (ii) entrepreneurs of the Foreign region make losses of capital borrowed from investors of the Home region: \( \tau_{f,h} R_f < r_h \). The necessary and sufficient conditions to observe an equilibrium with no cross-border capital flows are established in the following proposition.

**Proposition 3** Capital does not flow across regions at the equilibrium of the global capital market if and only if

\[
\frac{\tau_{h,f}}{\tau_f} < \frac{R_f}{R_h} < \frac{\tau_{f,h}}{\tau_{f,h}} \tag{11}
\]

**Proof.** Let us assume that capital does not flow across borders, the equilibrium cost of capital is then equal to the domestic return on capital in each region,

\[
r_h = \tau_h R_h \quad \text{and} \quad r_f = \tau_f R_f \tag{12}
\]

Moreover capital does not flow across borders when \( r_f > \tau_{h,f} R_h \) and \( r_h > \tau_{f,h} R_f \). Given the expression (12) for equilibrium costs of capital, these conditions are satisfied if and only if (11) holds.

A necessary condition for the equilibrium with no cross-border capital flows can therefore be written as \( \tau_f \tau_h > \tau_{h,f} \tau_{f,h} \). To provide a more transparent interpretation, let us assume that \( \tau_{i,j} \) can be divided into
two parts, one depending on the identity $i$ of the borrower $\alpha_i$ and one depending upon the identity $j$ of the investor $\beta_j$: $\tau_{i,j} = \alpha_i + \beta_j$ and $\tau_i = \alpha_i + \beta_i$. We can then derive the following corollary.

**Corollary 4** Writing $\tau_{i,j} = \alpha_i + \beta_j$ and $\tau_i = \alpha_i + \beta_i$ a necessary condition for the equilibrium with no cross-border capital flows to hold writes as

$$ (\alpha_h - \alpha_f) (\beta_h - \beta_f) < 0 $$  \hfill (13)

**Proof.** Assuming $\tau_{i,j} = \alpha_i + \beta_j$ and $\tau_i = \alpha_i + \beta_i$, the inequality $\tau_f \tau_h > \tau_{h,f} \tau_{f,h}$ can be easily simplified as (13). □

This necessary condition -for the no-cross border capital flow equilibrium- can be interpreted as a negative correlation across regions between efficiency in lending and efficiency in borrowing. If for instance, investors in the Home region are more efficient in lending than investors of the Foreign region, $\beta_h > \beta_f$ then this condition holds when entrepreneurs of the Home region are less efficient in borrowing than entrepreneurs of the Foreign region, $\alpha_h < \alpha_f$. Conversely the equilibrium with no cross-border capital flows can therefore be disregarded when efficiency in lending and efficiency in borrowing are positively correlated across regions. In this case, there is always an arbitrage opportunity for investors. Consider for instance the case where $\alpha_h < \alpha_f$ and $\beta_h < \beta_f$. Moreover to fix ideas assume that $R_h < R_f$. The Home region is hence less efficient than the Foreign economy both in terms of transaction costs and in terms of productivity. Then investors of the Home region always have incentives to lend to entrepreneurs of the Foreign region since the return on capital lent to entrepreneurs of the Foreign region $(\alpha_f + \beta_h) R_f$ is larger than the return $(\alpha_h + \beta_h) R_h$ on capital lent to entrepreneurs of the Foreign region. Conversely in the case where $\alpha_h > \alpha_f$ and $\beta_h > \beta_f$, investors of the Home region lend to entrepreneurs of the Foreign region if the gain associated with higher productivity is larger than the loss associated with lower efficiency in borrowing $(\alpha_h + \beta_h) R_h < (\alpha_f + \beta_h) R_f$. If this holds, then investors of the Home region will have incentives to lend abroad. If this does not hold, then investors of the Foreign region will have incentives to lend abroad because the gain stemming from a larger borrowing efficiency in the Home region compensates for the loss stemming from a lower productivity in the
Home region: \((a_h + \beta_f) R_h > (a_f + \beta_f) R_f\). The no cross border flows equilibrium is therefore impossible if condition (13) does not hold.

The necessary and sufficient condition (11) for the no-cross border capital flows implies that the ratio in the marginal productivity of capital \(R_f/R_h\) should be intermediate. For given \(\tau\), if the marginal return to capital in one region is much larger than the other then it is profitable for investors of the low productivity region to lend abroad. Consequently when \(R_f/R_h\) is not sufficiently large, there are no gains for investors of the Home region to lend to entrepreneurs of the Foreign region. Similarly, there are no gains for investors of the Foreign region to lend to entrepreneurs of the Home region when \(R_h/R_f\) is not sufficiently large. As a consequence when the ratio \(R_f/R_h\) is intermediate, there are no gains for investors to lending abroad and capital does not flow across borders. In the presence of transaction costs, if productivity differences are not sufficiently large then there are no gains to trade and the volume of cross border capital flows at the equilibrium is zero.

Figure 1: The no capital flow equilibrium
3.2 The one way capital flow equilibrium

The situation where capital flows exclusively from region \( i \) to region \( j \) is an equilibrium if and only if entrepreneurs of region \( j \) wish to borrow from investors of region \( i \) and entrepreneurs of region \( i \) wish to borrow from investors of region \( j \). This holds when two conditions are satisfied: (i) entrepreneurs of region \( j \) make profits on capital borrowed from investors of region \( i \): \( \tau_{j,i}R_j > r_i \) and (ii) entrepreneurs of region \( i \) make losses on capital borrowed from investors of region \( j \): \( \tau_{i,j}R_i < r_j \). The necessary and sufficient conditions under which this situation is an equilibrium are detailed in the following proposition.

**Proposition 5** Denoting \( \phi_{i,j} (\theta) = \tau_i + \tau_{i,j}\theta \), capital flows exclusively from region \( i \) to region \( j \) at the equilibrium of the global capital market if and only if

\[
\frac{R_j}{R_i} > \max \left\{ \frac{\tau_i}{\tau_{i,j}}, \frac{\phi_{i,j} (\theta_i)}{\phi_{i,j} (\theta_j)} \right\} \tag{14}
\]

**Proof.** When capital flows from region \( i \) to region \( j \), the equilibrium costs of capital \( r_i \) and \( r_j \) verify

\[
r_i = \tau_i R_i \quad \text{and} \quad r_j = (\tau_j + \tau_{j,i}\theta_j) R_j - \theta_j r_i \tag{15}
\]

Moreover capital flows from region \( i \) to region \( j \) if and only if \( r_j > \tau_{i,j}R_i \) and \( r_i \leq \tau_{j,i}R_j \). Given the expression (15) for equilibrium costs of capital \( r_i \) and \( r_j \), these conditions simplify as

\[
\frac{R_j}{R_i} > \max \left\{ \frac{\tau_i}{\tau_{i,j}}, \frac{\tau_{i,j} + \tau_i\theta_j}{\tau_j + \tau_{j,i}\theta_j} \right\}
\]

Since capital does not flow from region \( j \) to region \( i \), investors of region \( j \) lend all their capital \( k \) to entrepreneurs of region \( j \). Hence, entrepreneurs of region \( j \) can borrow \( \lambda_j + \theta_jk \) from investors of region \( i \). The volume of capital that flows from region \( i \) to region \( j \), \( k_{i,j} \) then writes as

\[
k_{j,i} = \lambda_j + \theta_jk
\]
The global capital market features one way capital flows when differences in productivity across regions are sufficiently large. Moreover, consistent with the standard approach, capital is more likely to flow to the region where entrepreneurs are the most productive. This is a standard result which however hides an important mechanism: When one region is more productive than the other, then entrepreneurs of the most productive region can afford to pay a larger cost of capital. As a result, investors of the less productive region have incentives to lend abroad and capital flows from the low to the high productivity region. This is the standard mechanism. Moreover the presence of complementarity between the different sources of capital introduces another mechanism: When entrepreneurs of the high productivity region borrow from abroad, the cost of capital they can afford to pay to domestic investors increases due to the complementarity between domestic and foreign capital. As a consequence, the cost of capital charged by investors of the high productivity region increases which further reduces incentives to borrow from abroad for entrepreneurs of the low productivity region.

![Figure 2: Equilibrium capital flows from the Foreign to the Home region](image-url)
3.2.1 Productivity and the direction of capital flows.

In the equilibrium where capital flows in one direction, it flows from the least productive to the most productive region if and only if the difference in productivity across regions is sufficiently large. Typically, capital flows from region $i$ to region $j$ at the equilibrium of the global capital market even if region $j$ is less productive when $R_j < R_i$ and (14) holds, i.e.

$$\max \left\{ \frac{\tau_{i,j}}{\tau_{j,i}}, \frac{\phi_{i,j}(\theta_j)}{\phi_{j,i}(\theta_i)} \right\} < \frac{R_j}{R_i} < 1 \quad (16)$$

We can then derive the following lemma as to the relationship between the direction of capital flows and the return to capital.

**Lemma 6** When capital flows in one direction at the equilibrium of the global market, capital cannot flow from the high productivity to the low productivity region if productivity and borrowing efficiency are positively related.

$$(\alpha_i - \alpha_j)(R_i - R_j) > 0$$

**Proof.** Given the decomposition $\tau_{i,j} = \alpha_i + \beta_j$, and given condition (16), capital flows to the low productivity region in the one way capital flow equilibrium if and only if

$$\max \left\{ \frac{\alpha_i - \alpha_j}{\alpha_j + \beta_i}, \frac{(\alpha_i - \alpha_j)(1 + \theta_j)}{\alpha_j + \beta_j + (\alpha_j + \beta_j)\theta_j} \right\} < \frac{R_j}{R_i} - 1 < 0$$

This condition cannot hold if $\alpha_i > \alpha_j$. Hence capital cannot flow from the high to the low productivity region if the low productivity region also exhibits low efficiency in borrowing. □

This lemma shows that if the low productivity region tends to be more efficient for borrowing than the high productivity region, then capital can flow uphill, i.e. from the high to the low productivity region. What typically happens in this case is that the low productivity of entrepreneurs is compensated by their large borrowing efficiency. Hence investors of the high productivity region may find it profitable to lend abroad, even if productivity abroad is lower. Practically, if we think of high productivity regions as developing
countries, and low productivity regions as developed countries, then productivity and borrowing efficiency are likely to be negatively related in which case capital can flow from high to low productivity countries. This result is however valid only as long as capital flows in one direction at the equilibrium of the global capital market. The next section examines whether this result still holds when the equilibrium of the global capital market is such that capital flows in both directions.

3.3 The two way capital flow equilibrium

The case where capital flows in both directions is an equilibrium if and only if entrepreneurs of the Home region wish to borrow from investors of the Foreign region and entrepreneurs of the Foreign region wish to borrow from investors of the Home region. This situation holds when two conditions are satisfied: (i) entrepreneurs of the Home region make profits on capital borrowed from investors of the Foreign region \( \tau_{h.f} R_h > r_f \) and (ii) entrepreneurs of the Foreign region make profits on capital borrowed from investors of the Home region \( \tau_{f.h} R_f > r_h \). Let us denote \( \theta = \min \{ \theta_h; \theta_{f}^{-1} \} \) and \( \theta' = \min \{ \theta_f; \theta_{h}^{-1} \} \). The following result can then be derived.

**Proposition 7** Denoting \( \phi_{i,j}(\theta) = \tau_{i} + \tau_{i,j} \theta \) and \( \varphi_{i,j}(\theta) = \tau_{i,j} + \tau_{i} \theta \) and assuming \( \theta_h \theta_f \neq 1 \), capital flows in both directions at the equilibrium of the global capital market if and only if

\[
\frac{\phi_{h.f}(\theta)}{\varphi_{f.h}(\theta)} < \frac{R_f}{R_h} < \frac{\phi_{h.f}(\theta')}{\varphi_{f.h}(\theta')}
\]

The cost of capital \( r_i \) charged by investors of region \( i \) then writes as

\[
r_i = \frac{\phi_{i,j}(\theta_i) R_i - \phi_{j,i}(\theta_j) \theta_i R_j}{1 - \theta_h \theta_f}
\]

The volume of capital \( k_{i,j} \) that flows from region \( j \) to region \( i \) writes as

\[
k_{i,j} = \frac{\mu_i - \theta_i \mu_j}{1 - \theta_h \theta_f}
\]
Proof. When capital flows in both directions then the equilibrium of the capital market in each region implies that gross interest rates $r_h$ and $r_f$ verify

$$r_h = (\tau_h + \tau_{h,f} \theta_h) R_h - \theta_h r_f$$
$$r_f = (\tau_f + \tau_{f,h} \theta_f) R_f - \theta_f r_h$$

Assuming $\theta_h \theta_f \neq 1$, this system has a unique solution which writes as (18). Capital flows in both directions at the equilibrium of the global capital market if and only if $\tau_{f,h} R_f > r_h$ and $\tau_{h,f} R_h > r_f$. Given expressions (18), when $\theta_f \theta_h < 1$, these conditions are met if and only if

$$\frac{\tau_{h,f} \theta_h + \tau_h}{\tau_f \theta_h + \tau_{f,h}} < \frac{R_f}{R_h} < \frac{\tau_{h,f} \theta_f + \tau_h}{\tau_f \theta_f + \tau_{f,h}}$$

On the contrary when $\theta_h \theta_f > 1$, capital flows in both directions at the equilibrium of the global capital market if and only if

$$\frac{\tau_{h,f} \theta_f + \tau_{h,f}}{\tau_{f,h} \theta_f + \tau_f} < \frac{R_f}{R_h} < \frac{\tau_{h,f} \theta_h + \tau_f}{\tau_{f,h} \theta_f + \tau_{f,h}}$$

Finally provided $k_h \leq k - \lambda_f$ and $k_f \leq k - \lambda_h$, the equilibrium of the capital market in the Home and the Foreign region write as

$$k_h + \lambda_f + \theta_f k_f = k$$
$$k_f + \lambda_h + \theta_h k_h = k$$

Determining the equilibrium quantities $k_i$ of capital lent domestically and the equilibrium quantities of capital $k_{i,j} = \lambda_i + \theta_i k_i$ that flows from region $j$ to region $i$ is then straightforward. ■

The necessary and sufficient condition (17) can be interpreted saying that productivity differences across regions should not be too large to observe an equilibrium where capital flows in both directions. This result is natural. In the presence of large productivity differences across regions, the low productivity region would be unable to attract capital from the high productivity region and capital would only flow from the low to the high productivity region. However as has been shown above, the necessary and sufficient condition (11) under which there are no capital flows at the equilibrium of the global capital market also implies
that productivity differences across regions should not be too large. Hence capital can either flow in both directions or not flow at all when entrepreneurs productivity is relatively similar across regions. The following corollary determines the precise conditions under which the equilibrium of the global capital market features two way capital flows or no capital flows at all.

**Corollary 8** Writing \( \tau_i = \alpha_i + \beta_i \) and \( \tau_{i,j} = \alpha_i + \beta_j \), a necessary condition to observe two way capital flows at the equilibrium of the global capital market is

\[
(\alpha_h - \alpha_f) (\beta_h - \beta_f) > 0
\]  

(20)

**Proof.** From (17), a necessary condition to observe two way capital flows at the equilibrium of the global capital market writes as \( \tau_{h,f} \tau_{f,h} > \tau_h \tau_f \). Moreover since we can write \( \tau_i = \alpha_i + \beta_i \) and \( \tau_{i,j} = \alpha_i + \beta_j \), the necessary condition \( \tau_{h,f} \tau_{f,h} > \tau_h \tau_f \) can be simplified as \( (\alpha_h - \alpha_f) (\beta_h - \beta_f) > 0 \). ■

A necessary condition for capital to flow in both directions across regions at the equilibrium of the global capital market is therefore the existence of a positive correlation between entrepreneurs efficiency in borrowing and investors efficiency in lending across regions. This condition can be interpreted saying that capital flows in both directions across regions only if one region dominates the other in terms of transaction costs for both lending and borrowing. In this case, capital effectively flows in both directions across regions if and only if the productivity of capital in the Home and the Foreign region are not too different from each other. On the contrary if no region dominates the other in terms of transaction costs on the capital market then there are no capital flows at the equilibrium of the global capital market when entrepreneurs’ productivity are not too different from each other across regions. To understand these results, let us consider the case where entrepreneurs of the Home region are less productive than entrepreneurs of the Foreign region. Then an entrepreneur of the Home region cannot profitably borrow from investors of the Foreign region unless the return to capital borrowed from investors of the Foreign region \( (\alpha_h + \beta_f) R_h \) is larger than the cost of
capital in the Foreign region \((\alpha_f + \beta_f) R_f\):

\[(\alpha_h + \beta_f) R_h > (\alpha_f + \beta_f) R_f\]

Given that \(R_h < R_f\) the difference in productivity \(\beta_f (R_f - R_h)\) must be sufficiently low compared to the difference in borrowing efficiency \(\alpha_h R_h - \alpha_f R_f\). Similarly, an entrepreneur of the Foreign region cannot profitably borrow from investors of the Home region unless the return to capital borrowed from investors of the Home region \((\alpha_f + \beta_h) R_f\) is larger than the cost of capital in the Home region \((\alpha_h + \beta_h) R_h\):

\[(\alpha_h + \beta_h) R_h < (\alpha_f + \beta_f) R_f\]

Given that \(R_h < R_f\) the difference in productivity \(\beta_h (R_f - R_h)\) must be sufficiently large compared to the difference in borrowing efficiency \(\alpha_h R_h - \alpha_f R_f\). As is clear these two conditions cannot be met together unless \(\beta_h > \beta_f\). Moreover the first condition - \((\alpha_h + \beta_f) R_h > (\alpha_f + \beta_f) R_f\) - implies that \(\alpha_h > \alpha_f\) since \(R_h < R_f\) by assumption.

![Figure 3: Equilibrium capital flows from Home to Foreign and from Foreign to Home](image-url)
Hence there cannot be cross border capital flows in both directions if the conditions $\beta_h > \beta_f$ and $\alpha_h > \alpha_f$ are not both satisfied, i.e. if there is not a positive correlation across regions between lending and borrowing efficiency.

4 Properties of the two way capital flow equilibrium

4.1 Indeterminacy in the direction of capital flows

When capital can flow in both directions across regions at the equilibrium of the global capital market, there may be multiple equilibria in the sense that the equilibrium with capital inflows and capital outflows may not be unique. The following proposition derives formally this possibility. i.e. $\tau_f \tau_h < \tau_{h,f} \tau_{f,h}$. We can then derive the following result.

Proposition 9 There are multiple equilibria on the global capital market if and only if $\theta_f \theta_h > 1$, $\tau_f \tau_h < \tau_{h,f} \tau_{f,h}$, and the ratio of productivity $R_f/R_h$ verifies

$$\frac{\tau_h \theta_f + \tau_{h,f}}{\tau_{f,h} \theta_f + \tau_f} < \frac{R_f}{R_h} < \frac{\tau_{h,f} \theta_h + \tau_h}{\tau_f \theta_h + \tau_{f,h}}$$

Proof. cf. appendix ■

The reason why there can be multiple equilibria on the global capital market is related to the positive externality entrepreneurs who borrowing from abroad exert on the cost of capital domestic investors can charge at the equilibrium. Consider for instance the case where entrepreneurs of the Home region do not borrow from investors of the Foreign region. Then the cost of capital investors of the Home region charge to their borrowers is relatively low; $r_h = \tau_h R_h$. As a result, it may be profitable for entrepreneurs of the Foreign region to borrow from investors of the Home region. In this case, the cost of capital investors of the Foreign region charge to their borrowers increases from $r_f = \tau_f R_f$ to $r_f = \tau_f R_f + \theta_f (\tau_{f,h} R_f - r_h)$. If the cost of capital $\tau_f$ is sufficiently large, then it is not profitable for entrepreneurs of the Home region to borrow from investors of the Foreign region and capital flows at the equilibrium from the Home to the
Foreign region. However the opposite situation can also hold and capital can flow in the opposite direction:

When entrepreneurs of the Foreign region do not borrow from investors of the Home region, the cost of capital investors of the Foreign region charge to their borrowers is relatively low; \( r_f = \tau_f R_f \). As a result, it may be profitable for entrepreneurs of the Home region to borrow from investors of the Foreign region. In this case, the cost of capital investors of the Home region charge to their borrowers increases from \( r_h = \tau_h R_h \) to \( r_h = \tau_h R_h + \theta_h (\tau_{h,f} R_h - \tau_f) \). If the cost of capital \( r_h \) is sufficiently large, then it is not profitable for entrepreneurs of the Foreign region to borrow from investors of the Home region and capital flows at the equilibrium from the Foreign to the Home region.

Finally if entrepreneurs of both the Home and the Foreign region borrow from abroad, then the cost of capital \( r_h \) and \( r_f \) will increase but not as much as to preclude entrepreneurs from borrowing from abroad. The direction of capital flows cannot be pinned down here because capital inflows from abroad exert a positive externality on the domestic cost of capital due to the complementarity between domestic and foreign borrowing.
4.2 Cross-border capital flows and the size of capital markets

When capital flows in both directions at the equilibrium, the volume of capital that flows across regions essentially depend on (i) parameters governing borrowing constraints $\lambda$ and $\theta$ and (ii) capital allocation $k$ within each region between entrepreneurs-borrowers and investors. Denoting $K_i$ investors in region $i$ capital endowment, the volume of capital that flows from region $j$ to region $i$ and the volume of capital that is intermediated in region $i$ write as

$$k_i = \frac{(K_i - \lambda_j) - \theta_j (K_j - \lambda_i)}{1 - \theta_i \theta_j} \text{ and } k_{i,j} = \lambda_i + \theta_i k_i$$

First, it straightforward to note that the equilibrium volume of capital $k_i$ (resp. $k_{i,j}$) entrepreneurs of region $i$ borrow from investors of region $i$ (resp. from investors of region $j$) increases with region $i$ investors’ capital supply $k$ if and only if $\theta_i \theta_f < 1$. To see it, consider an unit increase in the capital supply of investors in the Home region. Then, entrepreneurs of Home region need to increase domestic borrowing. Otherwise there would be an excess supply of capital since entrepreneurs of the Foreign region face a binding borrowing
constraint vis-à-vis investors of the Foreign region in equilibrium. When entrepreneurs of the Home region increase the volume of capital they borrow domestically by some amount $d$, they can borrow $\theta_h d$ extra units of capital from investors of the Foreign region. As a result, investors of the Foreign region need to cut lending to domestic entrepreneurs by $\theta_d d$ units of capital. Given complementarity in borrowing, entrepreneurs of the Foreign region need to cut by $\theta_f \theta_h d$ units of capital the volume of capital they borrow from investors of the Home region. Hence the change in the aggregate demand to investors of the Home region is $(1 - \theta_h \theta_f) d$ while their capital supply increases. As a consequence, entrepreneurs of the Home region borrow more from investors of the Home region ($d > 0$) when these investors are more wealthy if and only if $\theta_h \theta_f < 1$.

Second the increase in the volume of capital $k_i$ entrepreneurs of region $i$ borrow domestically is typically larger than the increase in the capital endowment $K_i$ of investors of region $i$. Hence region $i$ capital outflows decrease as the capital endowment $K_i$ of investors of region $i$ increases. Similarly, the equilibrium volume of capital $k_{i,j}$ that flows from region $j$ to region $i$ increases with region $i$ investors’ capital supply $k$ if and only if $\theta_h \theta_f < 1$. The reason for this result is fairly straightforward given (i) the complementarity between domestic borrowing $k_i$ and borrowing from abroad $k_{i,j}$ and (ii) the above property that domestic borrowing increases with the domestic capital supply $k$ if and only if $\theta_h \theta_f < 1$. This means that under the equilibrium with capital inflows and capital outflows, regions with larger capital markets -where $K$ is larger- are more likely, everything else equal, to be net capital importers as the volume of capital inflows is larger while the volume of capital outflows is lower.

This comparative statics exercise can also be extended to determine the impact of the parameters $\lambda$ and $\theta$ on the volumes of capital $k_h$ and $k_f$ intermediated domestically and on the volumes of cross border capital flows $k_{h,f}$ and $k_{f,h}$. As expected, when $\theta_h \theta_f < 1$, a larger parameter $\lambda_h$ raises the volume of capital that flows from the Home to the Foreign region. An increase $\delta$ in the parameter $\lambda_h$ raises the capital demand to investors of the Foreign region who need to reduce capital supply to domestic entrepreneurs by some amount $l$. Entrepreneurs of the Foreign region then need to reduce the volume of capital they borrow from

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4Remember that in equilibrium entrepreneurs make strictly positive profits on capital borrowed from abroad and strictly negative profits on capital borrowed domestically. As a result, entrepreneurs of the Home region can always outbid entrepreneurs of the Foreign region for capital supplied by lenders of the Foreign region. This is why lenders of the Foreign region would accept to cut domestic lending in favor of lending abroad.
investors of the Home region by an amount \( \theta_f l \). Investors of the Home region can then lend \( \theta_f l \) extra units of capital to domestic entrepreneurs who finally can raise the volume of capital they borrow from investors of the Foreign region by \( \theta_h \theta_f l \). Hence the net increase in the demand for capital from investors of the Foreign region is \( \delta - l (1 - \theta_h \theta_f) \). When \( \lambda_h \) increases (\( \delta > 0 \)), investors of the Foreign region tend to raise their capital supply to entrepreneurs of the Home region (\( l > 0 \)) if and only if \( \theta_h \theta_f < 1 \).

Finally a similar argument can be developed to understand the effect of a change in the parameters \( \theta_h \) and \( \theta_f \) on the pattern of capital allocation across regions. When entrepreneurs of the Home region experience an increase \( \Delta \theta \) in the parameter \( \theta_h \), they can borrow \( \Delta k_h \) extra units capital from investors of the Foreign region. Consequently, investors of the Foreign region need to cut lending to domestic entrepreneurs by some amount \( \Delta k_f \). However given the complementarity in borrowing constraints, the aggregate demand from entrepreneurs of the Home region to investors of the Foreign region increases by \( (\Delta \theta) k_h + (\Delta k_f) \theta_h \theta_f \) while the aggregate supply from investors of the Foreign region to entrepreneurs of the Home region increases by decreases by \( (\Delta k_f) \) units of capital. Hence investors of the Foreign region cut lending to domestic entrepreneurs by an amount \( \Delta k_f = \frac{\Delta \theta}{1 - \theta_h \theta_f} k_h \). This quantity is positive, i.e. entrepreneurs of the Home region raise borrowing from abroad when the complementarity index \( \theta_h \) increases, if and only if \( \theta_h \theta_f < 1 \). A region which benefits from an increase in its parameter \( \theta \) borrows more from abroad if and only if \( \theta_h \theta_f < 1 \).

### 4.3 The cost of capital and the income balance

Given the existence of borrowing constraints, there is no reason why the cost of capital should be equalized across regions. This implies that entrepreneurs of one region are paying more on their foreign liabilities. To get a proper assessment of the determinants of cross-border differences in the cost of capital, let us consider the case where the equilibrium with two way capital flows is unique, i.e. \( \theta_h \theta_f < 1 \). In this case given the expression (18) for equilibrium cost of capital \( r_i \) charged by investors of region \( i \), the cost to borrow from abroad is larger for entrepreneurs of the Foreign region, i.e. \( r_h > r_f \) if and only if

\[
\frac{R_f}{R_h} < \frac{1 + \theta_f \tau_{h,f} \theta_h + \tau_h}{1 + \theta_h \tau_{f,h} \theta_f + \tau_f}
\]
Moreover when $\theta_f \theta_h < 1$, the equilibrium with two way capital flows exists if and only if

$$\frac{\tau_{h,f} \theta_h + \tau_h}{\tau_f \theta_h + \tau_{f,h}} < \frac{R_f}{R_h} < \frac{\tau_{h,f} \theta_f + \tau_{h,f}}{\tau_{f,h} \theta_f + \tau_f}$$

We can derive the following result.

**Proposition 10** When there are two way capital flows at the equilibrium of the global capital market, the cost of capital charged by investors of the Home region is always larger than the cost of capital charged by investors of the Foreign region if and only if investors of the Home region are more efficient in lending than investors of the Foreign region: $\beta_h > \beta_f$.

**Proof.** Assuming $\theta_h \theta_f < 1$, the cost of capital charged by investors of the Home region is always larger than the cost of capital charged by investors of the Foreign region if and only if

$$\frac{\tau_{h,f} \theta_f + \tau_{h,f}}{\tau_{f,h} \theta_f + \tau_f} < 1 + \theta_f \frac{\tau_{h,f} \theta_h + \tau_h}{1 + \theta_h \tau_{f,h} \theta_f + \tau_f}$$

Under the assumption $\theta_h \theta_f < 1$ this condition simplifies as $\tau_h > \tau_{h,f}$ which is equivalent to $\beta_h > \beta_f$ given that $\tau_h = \alpha_h + \beta_h$ and $\tau_{h,f} = \alpha_h + \beta_f$. A similar argument can be made in the case $\theta_h \theta_f > 1$ which yields the same result.

The cost to borrow from abroad is always lower for entrepreneurs of the Foreign region if and only if investors of the Foreign region are more efficient than investors of the Home region. Put differently, the return to capital for investors is larger in the region where investors are more efficient. As a consequence, the cost to borrow is larger when lenders have a larger return. This result is interesting because it shows that when an equilibrium with capital inflows and outflows holds, then the ranking between the costs of capital is entirely driven by financial factors. Even with large productivity differences, the cost of capital charged by lenders of the most efficient region in lending is always larger and as a consequence, foreign liabilities of entrepreneurs of the least efficient region in lending are always more expensive.
4.4 Productivity shocks and the cost of capital

Expressions (18) shows that the cost for entrepreneurs to borrow from abroad increases with the return to capital abroad and decreases with the domestic return to capital when $\theta_h \theta_f < 1$. Hence when a region experiences an idiosyncratic positive productivity shock, the return on its foreign assets increases and the cost of its foreign liabilities decreases when $\theta_h \theta_f < 1$. Consider the case where the return to capital in the Home region increases. Then, this raises entrepreneurs of the Home region capital demand. Suppose entrepreneurs of the Home region raise the volume capital they borrow from investors of the Home region by one unit of capital. Then given the borrowing constraint they face vis-à-vis investors of the Foreign region, they can borrow $\theta_h$ extra units of capital from investors of the Foreign region. This implies that investors of the Foreign region need to reduce domestic lending -i.e. lending to entrepreneurs of the Foreign region- by $\theta_h$ units of capital. Given the borrowing constraint entrepreneurs of the Foreign region face vis-à-vis investors of the Home region, entrepreneurs of the Foreign region need to cut borrowing from investors of the Home region by $\theta_f \theta_h$ units of capital when they reduce domestic borrowing by $\theta_h$ units of capital. As a result the net change in the demand to investors of the Home region is $1 - \theta_f \theta_h$. When $\theta_f \theta_h < 1$, the demand for capital to investors of the Home region increases. Since supply is fixed, this translates into an increase in the interest rate $r_h$ investors of the Home region charge to their borrowers. Finally since the interest rate $r_f$ charged by investors of the Foreign region is negatively related at the equilibrium to the interest rate $r_h$ charged by investors of the Home region, the increase in the latter produces a decrease in the former.

We now examine the case of a global productivity shock.

**Corollary 11** When $\theta_h \theta_f < 1$, the cost for entrepreneurs of region $j$ to borrow from abroad increases when the return to capital faces a global positive shock if and only if investors lending efficiency $\beta_i$ verifies

$$\beta_i \geq \alpha_j (1 + \theta_j) - \alpha_i (1 + \theta_i) \frac{1}{1 - \theta_h \theta_f}$$
Proof. Let us consider that the return to capital in both region increases: $dR_h = dR_f = \delta > 0$. Then the change $dr_i$ in the cost for entrepreneurs of region $j$ to borrow from abroad writes as

$$dr_i = \frac{(\tau_i + \tau_{i,j}\theta_j) - (\tau_j + \tau_{j,i}\theta_i)}{1 - \theta_i\theta_j} \delta$$

Given that $\tau_{i,j} = \alpha_i + \beta_j$, the change $dr_i$ is positive if and only if

$$\beta_i \geq \frac{\alpha_j\theta_i(1 + \theta_j) - \alpha_i(1 + \theta_i)}{1 - \theta_i\theta_j}$$

The intuition for this result is related to the complementarity between lending efficiency $\beta$ and productivity $R$. When investors lending efficiency in region $i$ is high, i.e. $\beta_i$ is large, then a given increase in the return to capital $R_i$ produces a larger increase in the equilibrium cost of capital $r_i$ charged by investors of region $i$. Hence there are two countervailing forces. On the one hand, the equilibrium cost of capital $r_j$ charged by investors in region $j$ increases -given that the return to capital faces a global positive shock-, this tends to reduce the equilibrium cost of capital $r_i$ charged by investors of region $i$. On the other hand, the equilibrium cost of capital $r_i$ charged by investors in region $i$ increases following the increase in the return to capital and the more so the larger investors of region $i$ lending efficiency. The second (positive) effect tends to dominate the first (negative) effect when investors of region $i$ lending efficiency is relatively large. This property shows that a global positive shock to the productivity of capital tends to benefit to the most efficient investors but tends to hurt the least efficient investors. Moreover it is possible that the cost of capital decreases in all regions of the world economy after a global positive shock if investors efficiency is relatively low in all regions.

5 Conclusion.

To be drawn
References


